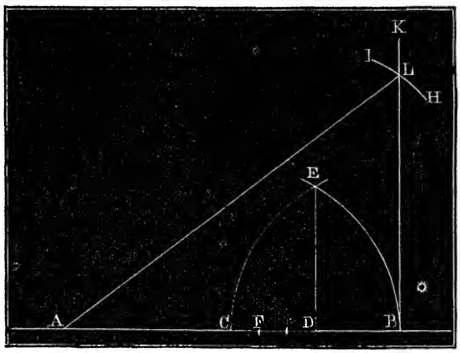
XXIII. "Approximate Geometrical Solutions of the Problems of the Duplication of the Cube and of the Quadrature of the Circle." By WILLIAM HAYDEN. Communicated by Prof. G. G. STOKES, Sec. R.S. Received November 30, 1871.

I. *Duplication of the Cube.*

Let A B be the given cube root. Erect the perpendicular B K; bisect A B in C, and with radius C B describe the arcs intersecting at E; let fall the perpendicular E D and trisect C D; then, with radius B F from E as a centre, describe the arc H I cutting B K in L; join A L, which will be nearly equal to the cube root of double the cube of A B, the amount of error being very small, which is proved as under.



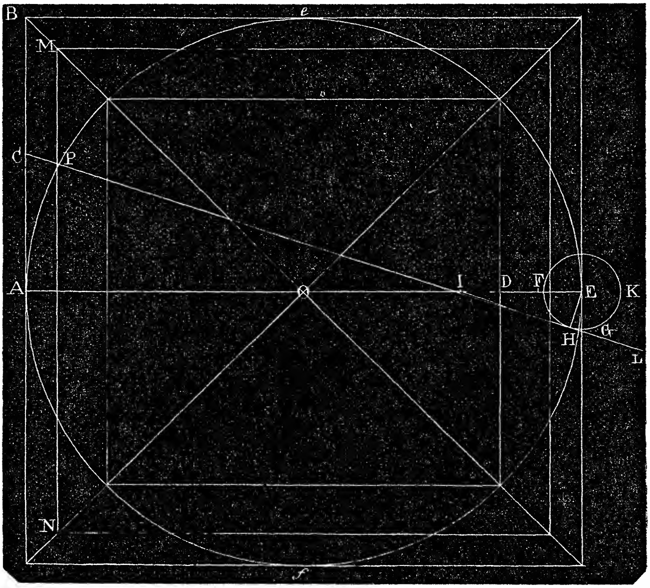
Let AB=3. Then

$$CB=1{\cdot}5,$$
$$BF=1{\cdot}25,$$
$$BD=\ {\cdot}75,$$
$$DE=\sin 60^\circ \times CB,$$
$$\sqrt[2]{(BF^2-BD^2)}=1,$$
$$(\sin 60^\circ \times CB)+1=BL,$$
$$\sqrt[2]{(AB^2+BL^2)}=AL=3{\cdot}7796264\ldots$$
$$\sqrt[3]{2}=1{\cdot}2599210$$
$$\tfrac{1}{3}AL=1{\cdot}2598754$$
$$\text{Error}\ldots\ {\cdot}0000456$$

II. *Quadrature of the Circle.*

In and about a given circle, as A *f* E *e*, draw the inscribed and tangent squares in the manner indicated by the figure, with their diagonals. Draw the diameter A E bisecting the opposite sides of the said squares; bisect A B in C, and D E in F; set off from E and D respectively E G and D I equal to D F, E F and join G I; from E let fall E H perpendicular to G I, and with radius E H describe the circle H K; draw the line C L touching

the circle H K, and cutting the circle A f E e in P; then through the point P, parallel to A B, draw M N terminating in its intersections with the



diagonals, as shown by the diagram, which will be one side of a square very nearly equal to the circle A f E e.

[The author has appended a mathematical calculation proving, in a somewhat indirect manner, that his construction gives an exceedingly close approximation. The construction leads directly to the following result:—

Let the radius of the circle be taken as unity, and let s be a side of the square given by the construction; then

$$s=\frac{\sqrt{3+4m+2m^2}-m}{1+m^2},$$

where

$$m=\frac{2+r\sqrt{17-4r^2}}{8-2r^2}, \quad r=\frac{3}{20}(\sqrt{10}-\sqrt{5})$$

(r is the radius of the circle H K, and m the tangent of the inclination of C L to A E).

Mr. Hayden has reduced these expressions to numbers, with the following result:—

$$r=\cdot 13893145240028844\ldots,$$
$$m=\cdot 322999477624\ldots,$$
$$s=1\cdot 7724538677\ldots,$$
$$s^2=3\cdot 141592713\ldots,$$

instead of $3\cdot 141592653\ldots$

—September. 1872 G. G. S.]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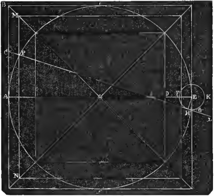

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